Program

	Monday 16 Sep	**Tuesday 17 Sep**	**Wednesday 18 Sep**	**Thursday 19 Sep**	**Friday 20 Sep**
9:00	Lecture T. Eisner	Lecture T. Eisner	Lecture W. Erb	Lecture B. Vedel	Lecture S. Neumayer
10:00	Break	Break	Break	Break	Break
10:30	Lecture B. Vedel	Lecture S. Neumayer	Lecture F. Nüske	Lecture F. Nüske	Lecture W. Erb
11:30	Lunch	Lunch	Lunch	Lunch	Lunch
 13:00 13:30 14:00	Session 1 S. Kunis F. Ba G. Maier	Session 3 D. Dũng N. Nagel R. Kunsch	Trip Hiking Tour	Session 5 D. Nuyens M. Kircheis TBA	End
14:30	Break	Break		Break	
 15:00 15:30 16:00	Session 2 M. Quellmalz N. Rux R. v. Rickenbach	Session 4 T. Pöschl M. Barthmann T. Sommerfeld	18:00 Buffet at restaurant Pelzmühle[1]	Session 6 Posters Posters Posters	
16:30	End	End		End	

[1] Pelzmühlenstraße 17, 09117 Chemnitz (also accessible by public transportation)

	Monday, 16 Sep
9:00	**T. Eisner** Ergodic Theorems
10:00	**Break**
10:30	**B. Vedel** Random Wavelet Series
11:30	**Lunch**
13:00	**S. Kunis** Sampling sparse graph signals
13:30	**F. Ba** Sparse additive function decompositions facing basis transforms
14:00	**G. Maier** On the Approximation of Gaussian Lipschitz Operators
14:30	**Break**
15:00	**M. Quellmalz** Motion Detection in Diffraction Tomography
15:30	**N. Rux** Wasserstein gradient flows for Riesz MMD regularized f-divergences
16:00	**R. von Rickenbach** Anisotropic Wavelet Matrix Compression of Integral Operators

	Tuesday, 17 Sep
9:00	**T. Eisner** Ergodic Theorems
10:00	**Break**
10:30	**S. Neumayer** Data-Driven Approaches for Solving Inverse Problems
11:30	**Lunch**
13:00	**D. Dũng** Optimal Gaussian-weighted quadrature for functions having Sobolev mixed smoothness
13:30	**N. Nagel** The L2-discrepancy of Latin hypercubes
14:00	**R. Kunsch** Monte Carlo quadrature with optimal confidence
14:30	**Break**
15:00	**T. Pöschl** Moving Least Squares Approximation on Spheres
15:30	**M. Barthmann** Vector-valued pointwise ergodic theorems for operators
16:00	**T. Sommerfeld** A Randomized Fast Algorithm for Frame Subsampling

	Wednesday, 18 Sep
9:00	**W. Erb** Computational Harmonic Analysis on Graphs
10:00	**Break**
10:30	**F. Nüske** Koopman-Based Learning for Stochastic Dynamical Systems
11:30	**Lunch**
13:00	**Trip**

	Thursday, 19 Sep
9:00	**B. Vedel** Random Wavelet Series
10:00	**Break**
10:30	**F. Nüske** Koopman-Based Learning for Stochastic Dynamical Systems
11:30	**Lunch**
13:00	**D. Nuyens** A comparison of lattice based kernel and truncated least squares approximations
13:30	**M. Kircheis** Direct inverse nonequispaced fast Fourier transforms
14:00	**TBA** TBA
14:30	**Break**
15:00	**Poster Session**

	Friday, 20 Sep
9:00	**S. Neumayer** Data-Driven Approaches for Solving Inverse Problems
10:00	**Break**
10:30	**W. Erb** Computational Harmonic Analysis on Graphs
11:30	**Lunch**
13:00	**End**

Invited Lectures

Tanja Eisner, University of Leipzig

Ergodic Theorems

Wolfgang Erb, Università degli Studi di Padova

Computational Harmonic Analysis on Graphs

Sebastian Neumayer, TU Chemnitz

Data-Driven Approaches for Solving Inverse Problems

Feliks Nüske, MPI Magdeburg

Koopman-Based Learning for Stochastic Dynamical Systems

Beatrice Vedel, Université de Bretagne Sud

Random Wavelet Series

Abstracts of Contributed Talks

Sampling sparse graph signals

Stefan Kunis
University Osnabrück

We study signals that are sparse either on the vertices of a graph or in the graph spectral domain. Recent results on the algebraic properties of random integer matrices as well as on the boundedness of eigenvectors of random matrices imply two types of support size uncertainty principles for graph signals. Indeed, the algebraic properties imply uniqueness results if a sparse signal is sampled at any set of minimal size in the other domain. The boundedness properties imply stable reconstruction by basis pursuit if a sparse signal is sampled at a slightly larger randomly selected set in the other domain.

Sparse additive function decompositions facing basis transforms

Fatima Antarou Ba
TU Berlin

High-dimensional real-world systems can often be well characterized by a small number of simultaneous low-complexity interactions. The analysis of variance (ANOVA) decomposition and the anchored decomposition are typical techniques to find sparse additive decompositions of functions. In this paper, we are interested in a setting, where these decompositions are not directly sparse, but become so after an appropriate basis transform. Noting that the sparsity of those additive function decompositions is equivalent to the fact that most of its mixed partial derivatives vanish, we can exploit a connection to the underlying function graphs to determine an orthogonal transform that realizes the appropriate basis change. This is done in three steps: we apply singular value decomposition to minimize the number of vertices of the function graph, and joint block diagonalization techniques of families of matrices followed by sparse minimization based on relaxations of the zero "norm" for minimizing the number of edges. For the latter one, we propose and analyze minimization techniques over the manifold of special orthogonal matrices. Various numerical examples illustrate the reliability of our approach for functions having, after a basis transform, a sparse additive decomposition into summands with at most two variables.

On the Approximation of Gaussian Lipschitz Operators

Gregor Maier
University of Bonn

Over the past few years, operator learning – the approximation of mappings between infinite-dimensional function spaces using ideas from machine learning – has attracted increased research attention. Approximate operators, learned from data, hold promise to serve as efficient surrogate models for problems in scientific computing. Multiple model designs have been proposed so far and their efficiency has been demonstrated in various practical applications. The empirical findings are supported by a (slowly) growing body of theoretical approximation garantuees. The latter focus to a large extent on linear and holomorphic operators. However, far less is known about the approximation of (nonlinear) operators which are merely Lipschitz continuous.

In this talk, I will focus on Lipschitz operators in a Gaussian setting. I will first consider their polynomial approximation by Hermite polynomials and present lower and upper bounds on the best s-term error. This will be followed by a discussion on the approximation of Lipschitz operators by arbitrary (adaptive) sampling algorithms, which will result in sharp error bounds. Finally, I will conclude by also addressing the problem of recovering Lipschitz operators from i.i.d. pointwise samples.

This is joint work with Ben Adcock (SFU).

Motion Detection in Diffraction Tomography

Michael Quellmalz
TU Berlin

We study the mathematical imaging problem of optical diffraction tomography for the scenario of a rigid particle rotating in a trap created by acoustic or optical forces. Under the influence of the inhomogeneous forces, the particle carries out a time-dependent smooth, but irregular motion. The rotation axis is not fixed, but continuously undergoes some variations, and the rotation angles are not equally spaced, which is in contrast to standard tomographic reconstruction assumptions. Once the time-dependent motion parameters are known, the particle's scattering potential can be reconstructed based on the Fourier diffraction theorem, considering it is compatible with making the first order Born or Rytov approximation.

The aim of this presentation is twofold: We first need to detect the motion parameters from the tomographic data by detecting common circles in the Fourier-transformed data. This can be seen as analogue to method of common lines from cryogenic electron microscopy (cryo-EM), which is based on the assumption that the assumption that the light travels along straight lines. Then we can reconstruct the scattering potential of the object utilizing non-uniform Fourier methods.

This is joint work with Peter Elbau, Clemens Kirisits, Otmar Scherzer, Eric Setterqvist and Gabriele Steidl.

Wasserstein gradient flows for Riesz MMD regularized f-divergences

Nicolaj Rux
TU Berlin

In generative models Wasserstein gradient flows can be used to generate samples. Symmetric positive definite bounded universal kernels $K : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}$ induce a metric on the space of measures called Maximum Mean Discrepancy (MMD). Another natural distance on measures is the KL divergence. In recent works, new distances were constructed by interpolating between the more general f-divergences and the MMD. These new distances have a close connection to the classical theory of Moreau envelopes from Convex Analysis. We extend the existing theory to a modification of the unbounded Riesz kernel $R(x, y) = \|x\| + \|y\| - \|x - y\| + 1$ and show, that this distances with the MMD of the unbounded Riesz kernel is also lower semi continuity and compute its Fenchel conjugate. This is joint work with Sebastian Neumayer, Viktor Stein and Gabriele Steidl.

Anisotropic Wavelet Matrix Compression of Integral Operators

Remo von Rickenbach
University of Basel

Consider an integral operator equation $\mathcal{L}u = f$ posed on the unit square $\square := [0,1]^2$ or a smooth manifold $\Gamma \subset \mathbb{R}^3$. It is assumed that $\mathcal{L} : H^q \to H^{-q}$ is continuous and uniformly elliptic, where for sufficiently smooth u there holds

$$\mathcal{L}u(\boldsymbol{x}) = \int \kappa(\boldsymbol{x}, \boldsymbol{y}) u(\boldsymbol{y}) \, \mathrm{d}\boldsymbol{y}.$$

In particular, we assume that the kernel κ is asymptotically smooth of order $2q$, that is,

$$\left| \partial_{\boldsymbol{x}}^{\boldsymbol{\alpha}} \partial_{\boldsymbol{y}}^{\boldsymbol{\beta}} \kappa(\boldsymbol{x}, \boldsymbol{y}) \right| \lesssim \|\boldsymbol{x} - \boldsymbol{y}\|^{-(2+2q+|\boldsymbol{\alpha}|+|\boldsymbol{\beta}|)}, \quad 2 + 2q + |\boldsymbol{\alpha}| + |\boldsymbol{\beta}| > 0.$$

This setting covers the practically important case of boundary element methods.

Given a wavelet basis $\Psi = \{\psi_\lambda : \lambda \in \nabla\}$ of H^q, we ask the following question: How many basis functions are necessary to approximate the unknown solution u up to a given precision ε? In other terms, what is the smallest N such that there exists a finite index set $\Lambda \subset \nabla$ with $|\Lambda| \leq N$ and $\|u - u_\Lambda\|_{H^q} \leq \varepsilon$?

In this talk, we will charaterise the function spaces which can be approximated with N terms at the rate N^{-s} if the underlying basis set Ψ is of *anisotropic* nature. Moreover, we will discuss under which circumstances the solution u of the operator equation $\mathcal{L}u = f$ can be approximated at the same rate as if full knowledge on the function u was provided. Therefore, we will have a brief look at the concept of $s^\star$-*compressibility* and the difficulties arising from the anisotropic structure of the wavelet functions.

Optimal Gaussian-weighted quadrature for functions having Sobolev mixed smoothness

Dinh Dũng
Vietnam National University, Hanoi

This lecture presents some recent results on optimal Gaussian-weighted quadrature for functions on $\mathbb{R}^d$ belonging to the Gaussian-weighted Sobolev space $W^r_p(\mathbb{R}^d, \gamma)$ of mixed smoothness $r \in \mathbb{N}$ and for $1 \leq p < \infty$, where γ is the standard Gaussian measure. In the high dimensional case ($d > 1$), we prove the right convergence rate of optimal quadrature for $1 < p < \infty$ and upper bound of optimal quadrature for $p = 1$, and propose novel methods for constructing optimal quadratures based on sparse grids. In the one-dimensional case ($d = 1$), we obtain the right convergence rate of optimal quadrature for $1 \leq p < \infty$. For detail see [1,2].

[1] D. D ung, *Numerical weighted integration of functions having mixed smoothness*, J. Complexity **78**(2023) 101757.

[2] D. Dũng and V. K. Nguyen, *Optimal numerical integration and approximation of functions on $\mathbb{R}^d$ equipped with Gaussian measure*, IMA Journal of Numerical Analysis **44**(2024), 1242–1267.

The L2-discrepancy of Latin hypercubes

Nicolas Nagel
TU Chemnitz

As was previously observed by Hinrichs and Oettershagen, point sets constructed from permutations seem to give approximate global optimizers of L2-discrepancy in the square. Inspired by this, we study general point sets derived from permutations. We prove a formula that was first observed by Hinrichs, Kritzinger and Pillichshammer for certain point sets and generalize it first to permutations and then to certain Latin hypercube point sets in higher dimension. These formulas allow for an easy analysis of the optimal L2-discrepancy of this notion of Latin hypercubes that we also want to mention. This talk also aims to give an accessible introduction to L2-discrepancy in general.

Monte Carlo quadrature with optimal confidence

Robert Kunsch
RWTH Aachen

We study the numerical integration of smooth functions using finitely many function evaluations within randomized algorithms, aiming for the smallest possible error guarantees with high probability (the so-called confidence level). There are different strategies for constructing robust estimators, for example, taking the median of several repeated realizations of a basic method where the number of required repetitions depends on the desired confidence level. For Sobolev classes of continuous functions, however, we can find linear integration methods that have optimal error bounds for all confidence levels. Numerical experiments show that the tails of the error distribution are significantly smaller compared to previously known methods.

Moving Least Squares Approximation on Spheres

Tim Pöschl
Freiberg University of Mining and Technology

The moving least squares (MLS) approximation is a form of scattered data approximation. Let $f\colon \mathbb{S}^{d-1} \to \mathbb{R}$ be a function from the $d-1$ dimensional sphere to $\mathbb{R}$ with known values at the nodes $y_1, \ldots, y_N \in \mathbb{S}^{d-1}$. Given an evaluation point $y \in \mathbb{S}^{d-1}$, the MLS approximation $\mathcal{M}f(y) \approx f(y)$ is obtained by first computing a local least squares approximation g^* of f using only a local subset of all nodes y_i, and then setting $\mathcal{M}f(y) \coloneqq g^*(y)$.

Using spherical harmonics up to degree $L \in \mathbb{N}$ as an ansatz space yields for functions $f \in \mathcal{C}^{L+1}(\mathbb{S}^{d-1})$ the approximation order $\mathcal{O}(h^{L+1})$, where h denotes the fill distance of the sampling nodes y_i.

We show that the dimension of the ansatz space can be almost halved, by including only spherical harmonics of even or odd degree up to L, while preserving the same order of approximation.

Vector-valued pointwise ergodic theorems for operators

Micky Barthmann
TU Chemnitz

The pointwise ergodic theorem of Birkhoff has been generalized in many directions. One direction of generalization has been to consider linear operators that are more general than Koopman operators, such as Dunford-Schwartz operators, i.e., $L^1 - L^\infty$ contractions on a σ-finite measure space. Another direction of generalization is to consider (finite) measure preserving systems that have stronger mixing properties than ergodicity, as was done for example in the Wiener-Wintner Theorem. I will discuss a uniform vector-valued Wiener-Wintner Theorem for a class of operators that includes compositions of ergodic koopman operators and contractive multiplication operators. This is based on joint work with Sohail Farhangi.

A comparison of lattice based kernel and truncated least squares approximations

Dirk Nuyens
KU Leuven

We consider multivariate functions which can be written as absolutely converging Fourier series belonging to a function space with a given smoothness. We assume standard informa- tion: we can evaluate the function in n points. The smoothness of the function space now determines how fast such an approximation converges in terms of n.

One way is to approximate the Fourier coefficients on a truncated index set using an n point lattice rule. Since lattice rules are particularly well suited for approximating integrals in such a function space, they might appear as a good choice. However, it is known that this only converges at half the speed. On the bright side, all calculations can be done at FFT speed, even when one wants a sequence of approximations.

One could also determine these Fourier coefficients by a least squares method. There are very nice results that this method allows to achieve the optimal speed of convergence for the function approximation problem. It is even possible to get hold of a good point set, with high probability, by just drawing random points. On the negative side, the matrix appearing in the least squares problem does not have any structure, but one could subsample a lattice point set to regain the FFT structure.

A third method is to use a kernel approximation which is widely used in statistical sciences. It can be shown that, for a given point set, the kernel approximation achieves the best worst case error compared to all possible other algorithms using the same function values. In combination with a lattice point set, the kernel approximation can also be computed at FFT speeds. However, the same problem with the convergence speed appears.

Direct inverse nonequispaced fast Fourier transforms

Melanie Kircheis
TU Chemnitz

The well-known discrete Fourier transform (DFT) can easily be generalized to arbitrary nodes in the spatial domain. The fast procedure for this generalization is referred to as nonequispaced fast Fourier transform (NFFT). Various applications such as MRI, solution of PDEs, etc., are interested in the inverse problem, i. e., computing the Fourier coefficients of a trigonometric polynomial from given nonequispaced sampling data. In contrast to iterative procedures, where multiple iteration steps are needed for computing a solution, we focus especially on so-called direct inversion methods. More precisely, we introduce a new density compensation scheme that leads to an exact reconstruction for trigonometric polynomials, and propose a matrix optimization approach using Frobenius norm minimization to obtain an inverse NFFT.

Furthermore, since most applications are even concerned with the analogous continuous problem, i. e., the reconstruction of point evaluations of the Fourier transform from given measurements of a bandlimited function, we study the extension properties of the previous techniques to this setting.