Modal analysis in mixed finite element methods

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Modal analysis is a widely used approach to approximate solutions of time dependent problems of continuum mechanics. The equilibrium equations are transformed into an eigenvalue problem in which the eigenfunctions are the amplitudes and the eigenvalues are the squares of the frequencies. In the case of symmetric and positive definite operators such as elasticity, a complete set of eigenfunctions and associated positive eigenvalues exist, and therefore the true solution can be expressed as a series of modes. When mixed formulations are required as in the case of incompressible elasticity, and spatial discretisations based on the Galerkin variational form for the class of saddle-point problems are used, it is well known that the approximations are optimally convergent if the finite element spaces for different fields satisfy suitable inf-sup conditions. The stability issues when classical mixed methods are used with arbitrary interpolations violating this condition can be avoided by numerous stabilisation techniques. These apply to both the stationary boundary value problems and the associated eigenvalue problems. An extra care is needed when the latter is to be approximated by a residual based approach which may lead to a quadratic problem even if the original one is linear.

In this presentation we first describe a mixed finite element formulation for such eigenvalue problems that preserve the linearity of the continuous problem and can be solved using arbitrary interpolations for the unknown fields. The approach is mainly based on the variational multiscale concept which assumes that the unknown can be split into a finite element component and a subgrid scale to be modelled. The key point is to consider that this subgrid scale is orthogonal to the finite element space. Next, we present the extension of modal analysis for elastic materials and show that each pair (amplitude and frequency) can be obtained from an eigenvalue problem that can be split into the finite element scale and the subgrid scale. A set of solution pairs of this eigenvalue problem are computed, and finally the time approximation to the continuous solution is obtained taking a few modes, those associated with higher energy.

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