Constrained L^p Approximation of Shape Tensors and its Relation to Shape Gradients

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The recent pioneering work by Deckelnick, Herbert and Hinze [1] enables the approximation of the shape gradient with respect to $W^{1,\infty}$, the natural norm for the space of shape deformations in this context. An alternative route towards such shape gradients is investigated in [3] via the constrained L^p best approximation of shape tensors. The shape gradient appears as Lagrange multiplier in the corresponding optimality system. Shape tensors were introduced earlier, with a different motivation, by Laurain and Sturm in [2]. We review the main result from [3] on the relation between the above best approximation in L^p ($p \in (1,2]$) and the shape gradient in W^{1,p^*} ($1/p + 1/p^* = 1$). Moreover, we generalize this result to the symmetric best approximation of shape tensors which is related to an elasticity-type norm. The numerical realization by lowest-order Raviart-Thomas elements for the shape tensor approximation is also presented and tested for several shape optimization problems of variable difficulty including state-of-the-art test problems.

References:

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