

phi-FEM: an optimally convergent finite element method on unfitted meshes

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We shall report on some new developments concerning ϕ -FEM – a fictitious domain finite element method. The method is well suited for elliptic and parabolic problems posed in domains given by smooth level-set functions without constructing a mesh fitting the boundary. ϕ -FEM achieves optimal convergence (attested both theoretically and numerically) using the finite elements of any order. Unlike other recent fictitious domain-type methods (XFEM, CutFEM), our approach does not need any non-standard numerical integration, neither on the cut mesh elements nor on the actual boundary. The main idea of ϕ -FEM is to extend the original problem slightly outside its domain (namely to the domain occupied by all the mesh cell having a non-empty intersection with the physical domain), and then write down a (formal) variational formulation on the extended domain using the level set to impose the boundary conditions “implicitly” (either via a product of the levelset with the finite elements or by introducing auxiliary variables).

References:

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