Stokes equation with non smooth boundary data

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For the Dirichlet Boundary Control of the Stokes equation, it must first be discussed how to understand the solution of the Dirichlet problem for the Stokes equations when the Dirichlet data is not smooth, i.e. when it is only in $L^2(\Omega)$.

A weak solution $(y, p) \in H^1(\Omega)^2 \times L^2(\Omega)$ cannot be expected. Instead, the very weak formulation is considered and existence and uniqueness results are given. In addition, the regularity of the solution is discussed, when the boundary data is in $H^{\frac{1}{2}-s}(\Omega)$ with $s \in [0, 1]$. For the discretization, the regularization approach is considered and it is shown how to correctly handle the compatibility condition $\int u \cdot n = 0$. Discretization error estimates are presented and validated by numerical examples.

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