

# Pressure-Robustness, an Elephant in the Room, and a Conceptual Review for Mixed Finite Element Methods

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In recent years, the notion of pressure-robustness aimed at overcoming an “elephant in the room” in the discretization theory of inf-sup stable mixed finite element methods for incompressible fluid dynamics. Originally coined “poor mass conservation”, the issue provoked some headache in theory and practice since the 1980ies, and has been traced back finally to an inappropriate discrete treatment of gradient fields, which lie in the kernel of a semi norm quantifying the strength of data driving the flow dynamics. Subsequently, pressure-robust flow discretizations have been constructed, which allow for error estimates that are qualitatively different to the classical ones presented in the influential textbook by Girault and Raviart. Moreover, not only error estimates, but also basic concepts of mixed methods have been reviewed in the research on pressure-robustness, This concerns the notion of data, the appropriate conformity, the used model problems, and even the notion of locking, which is central for classical mixed finite element theory. Some history and various numerical examples illustrate the findings.

## References:

- [1] <https://smaj-jcm.centre-mersenne.org/articles/10.5802/smai-jcm.44/>
- [2] <https://epubs.siam.org/doi/abs/10.1137/15M1047696>
- [3] <https://epubs.siam.org/doi/abs/10.1137/20M1351230>
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