Weak maximum principle of finite element methods for parabolic equations

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Maximum principle is a fundamental mathematical tool to study elliptic and parabolic partial differential equations. The discrete counterpart of maximum principle associated to finite element methods has a long history of research and remains an active field. Unlike its continuous counterpart, the discrete maximum principle is not an inherent property and is significantly influenced by the triangulation of the physical domain. In dimensions three and higher, it becomes challenging to assure the discrete maximum principle for even piecewise linear elements. Nonetheless, a large number of applications do not require a strong discrete maximum principle. In 1980, A. Schatz demonstrated that a weak maximum principle (also known as the Agmon-Miranda principle) is applicable to a broad spectrum of finite elements on general quasi-uniform triangulation in any two-dimensional polygonal domains, which was later extended to three dimensions. The situation for parabolic equations is more complex, depending not only on space discretization but also on time discretization. In my talk I will review the history of the weakened strong discrete maximum principle and show new results that establish the weak maximum principle of finite element methods for parabolic equations semidiscrete and fully discrete Galerkin finite element solutions.

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