Properties of the Iterated penalty method for Scott-Vogelius elements

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Using the Scott-Vogelius mixed finite element spaces for solving the Stokes equations yields exactly divergence free velocity approximations. Therefore, the method is pressure-robust. The Scott-Vogelius velocity solution can be approximated by the iterated penalty method (IPM). This is an Uzawa-type iteration that does not require an explicit local basis of the pressure space.

We propose a local mesh modification that removes nearly singular vertices. By this we obtain convergence in few iterations and high accuracy in both the velocity and pressure approximation. It is well known that the velocity divergence converges to zero geometrically, but we also prove that it always decreases monotonically. Furthermore, we present a quasi-optimality estimate for the velocity approximation in each step of the IPM iteration which quantifies the influence of the pressure best approximation error. All of this can be done for inhomogeneous boundary conditions.

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