On full linear convergence and optimal complexity of adaptive FEM with inexact solver

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The ultimate goal of any numerical scheme for partial differential equations (PDEs) is to compute an approximation of user-prescribed accuracy at quasi-minimal computational time. To this end, the algorithmic realization of a standard adaptive finite element method (AFEM) integrates an inexact solver and nested iterations with discerning stopping criteria to balance the different error components. The analysis ensuring optimal convergence order of AFEM with respect to the overall computational cost critically hinges on the concept of R-linear convergence of a suitable quasi-error quantity. This talk presents recent advances in the analysis of AFEMs to overcome several shortcomings of previous approaches. First, a new proof strategy with a summability criterion for R-linear convergence allows to remove typical restrictions on the stopping parameters of the nested adaptive algorithm. Second, the usual assumption of a (quasi-)Pythagorean identity is replaced by the generalized notion of quasi-orthogonality from [Feischl, Math. Comp., 91 (2022)]. Importantly, this paves the way towards extending the analysis to general inf-sup stable problems beyond the energy minimization setting. Numerical experiments investigate the choice of the adaptivity and stopping parameters.

References:

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