Analysis of Divergence-Preserving Unfitted FEM for the Mixed Poisson Problem

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Geometrically unfitted finite element methods, e.g. CutFEM, XFEM, or unfitted DG methods, are popular modern tools for the numerical approximation of partial differential equations on complex or moving geometries, allowing the separation of the geometry description from the computational mesh. Commonly, these methods rely on stabilization techniques, for instance, ghost penalty (GP) stabilization, that ensure stability independent of the local cut configuration. Mixed finite element methods based on special vectorial finite element spaces, e.g. H(div)-conforming finite element spaces, however, are tailored to preserve conservation properties (e.g. mass conversation) exactly on the discrete level. While adding stabilization terms ensures the stability of the problem, the conservation properties are perturbed. In this talk, we introduce and analyze a stable discretization of the unfitted mixed Poisson problem with Dirichlet boundary conditions. Notably, our approach does not require stabilization terms that pollute the mass balance. The key idea is to formulate the divergence constraint on the active mesh instead of the physical domain, which yields a robust discretization independent of the cut configuration without the need for stabilization. This modification does not affect the accuracy of the flux variable and by applying postprocessing strategies to the scalar variable, we achieve optimal convergence rates for both variables and even superconvergence for the scalar variable.

References:

[1] https://arxiv.org/abs/2306.12722

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