Low-order non-conforming finite elements for the 3D Stokes equations

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This talk is based on [1] and proposes two low-order nonconforming finite element methods (FEMs) for the three-dimensional Stokes flow that generalize the nonconforming FEM of Kouhia and Stenberg [2]. The finite element spaces proposed in this talk consist of two globally continuous components (one piecewise affine and one enriched component) and one component that is continuous at the midpoints of interior faces. We prove that the discrete Korn inequal- ity and a discrete inf–sup condition hold uniformly in the mesh size and also for a nonempty Neumann boundary. Based on these two results, we show the well-posedness of the discrete problem. Two counterexamples prove that there is no direct generalization of the Kouhia–Stenberg FEM to three space dimen- sions: the finite element space with one nonconforming and two conforming piecewise affine components does not satisfy a discrete inf-sup condition with piecewise constant pressure approximations, while finite element functions with two nonconforming and one conforming component do not satisfy a discrete Korn inequality.

References

[1] R. Kouhia and R. Stenberg. A linear nonconforming finite element method for nearly incompressible elasticity and Stokes flow. Comput. Methods Appl. Mech. Engrg., 124(3):195-212, 1995

[2] J. Hu and M. Schedensack. Two low-order nonconforming finite element methods for the Stokes flow in 3D. IMA J. Numer. Anal., 39(3):1447-1470, 2019