A local discontinuous Galerkin method based on L1-2-3 formula for two-dimensional subdiffusion equations

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The problem considered here is the two-dimensional time-fractional (subdiffusion) equation with the Caputo fractional derivative of order $\alpha \in (0, 1)$, which is approximated usually by using some formulae known as L1 and L1-2. These formulae are obtained, respectively, by applying the linear and quadratic Lagrange interpolation method. Based on the cubic Lagrange interpolation, a higher-order recipe called L1-2-3 has been constructed successfully in [1]. This new formula has been applied in solving subdiffusion equations besides the finite difference method and finite element method in [2, 3]. Furthermore, approximating the Caputo fractional derivative with such formulae is an ongoing topic, see e.g., [4].

On the other hand, a local discontinuous Galerkin (LDG) method for 2D time-fractional diffusion equations has been constructed and analyzed in [5]. This numerical method is based on an LDG method in space and the L1 or L1-2 schemes in time. We aim to proceed to construct a numerical method, which is based on an LDG method in space and the L1-2-3 formula in time. Then, we investigate the method's numerical stability and convergence for both rectangular and triangular meshes and show that the method is unconditionally stable. Numerical results indicate the effectiveness and accuracy of the algorithm and confirm the analysis.

References

[1] R. Mokhtari, F. Mostajeran, A high order formula to approximate the Caputo fractional derivative. Commun. Appl. Math. Comput. 2, 1-29, 2020.

[2] R. Mokhtari, M. Ramezani, G. Haase, Stability and convergence analyses of some difference schemes for solving the subdiffusion equation, Submitted.

[3] M. Ramezani, R. Mokhtari, G. Haase, Application of some high-order numerical formulae in solving time-fractional diffusion equations, 32nd Chemnitz FE Symposium, Germany, 2019.

[4] M. Ramezani, R. Mokhtari, G. Haase, Some high order formulae for approximating Caputo fractional derivatives, Appl. Numer. Math. 153, 300-318, 2020.

[5] S. Yeganeh, R. Mokhtari, J. S. Hesthaven, A local discontinuous Galerkin method for two-dimensional time fractional diffusion equations. Commun. Appl. Math. Comput. `https://doi.org/10.1007/s42967-020-00065-7`, 2020.