Bayesian Inversion: Posterior Sampling using Surrogate Models

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We aim at the use of the Bayesian inverse approach for the identification of parameters of differential equations. As unknown parameters, we can consider boundary conditions, source or material parameters in a domain of interest. The solution of such identification problems typically involves repetitive evaluations of a corresponding discretized boundary value problem using numerical methods, e.g. the finite element method (FEM) or its variants. These computational demands can be reduced using surrogate models.

The result of the Bayesian inversion is the estimation of the posterior distribution, i.e. the joint probability distribution of the vector of unknown parameters. Since the posterior probability density function depends on the solution of some parametric boundary value problem, it cannot be expressed analytically or sampled directly. The basic Metropolis- Hastings (MH) algorithm can be used to provide samples from the posterior distribution; however, the sampling process is too costly due to high number of evaluations of the forward problem. We shall show that the delayed acceptance MH algorithm combined with a suitable surrogate model works and reduces substantially the number of required forward problem evaluations. We examined several approaches for the surrogate model construction: the stochastic Galerkin method, the stochastic collocation method, and radial basis functions. This approach was tested on model problems, numerical experiments include the tuning of settings of the sampling algorithm parameters.

References:

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